$\square$ Code No. : 11221 N/O
VASAVI COLLEGE OF ENGINEERING (AUTONOMOUS), HYDERABAD
Accredited by NAAC with A++ Grade
B.E. I-Semester Main \& Backlog Examinations, Jan./Feb.-2024

Calculus \& Linear Algebra
(Common for CSE, AIML \& IT)
Time: $\mathbf{3}$ hours
Max. Marks: 60
Note: Answer all questions from Part-A and any FIVE from Part-B
Part-A $(10 \times 2=20$ Marks $)$

| Q. No. | Stem of the question | M | L | CO | PO |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | Define curvature | 2 | 1 | 1 | 1,12 |
| 2. | Find the Taylor's series expansion of $\operatorname{Sin} \mathrm{x}$ about the point $\mathrm{x}=\frac{\pi}{2}$ | 2 | 2 | 1 | 1,12 |
| 3. | If $x y^{2} \cos x y+x^{2} y \sin x y=0$, find $\frac{d y}{d x}$. | 2 | 2 | 2 | 1,12 |
| 4. | If $U=f(x, y), x=G(s, t), y=H(s, t), s=g(r)$ and $t=h(r)$, then find the total derivative of $U$ with respect to $r$. | 2 | 3 | 2 | 1,12 |
| 5. | Is the set of all real numbers R over the Complex numbers C , with usual complex multiplication as scalar multiplication, a vector space? Explain | 2 | 4 | 3 | 1,2,12 |
| 6. | Define the Dimension of a vector space V(F) | 2 | 2 | 3 | 1,12 |
| 7. | Define Null space | 2 | 1 | 4 | 1,12 |
| 8. | State Rank-Nullity Theorem. | 2 | 1 | 4 | 1,12 |
| 9. | When a matrix of the linear transformation diagonalizable.? | 2 | 2 | 5 | 1,12 |
| 10. | If $x=(2,1+i, i), y=(2-i, 2,1+2 i)$ be two elements of the inner product space $C^{3}(\mathrm{C})$ with respect to the standard inner product, then find $\langle x, y\rangle$. | 2 | 3 | 5 | 1,12 |
|  | Part-B $(5 \times 8=40 \mathrm{Marks})$ |  |  |  |  |
| 11. a) | Find the radius of curvature at the origin of the curve $y^{2}=x^{2} \frac{(a+x)}{(a-x)}$ | 4 | 2 | 1 | 1,12 |
| b) | Find the Taylor's series expansion of $f(x)=\frac{1}{1+x}$ about $\mathrm{x}=1$ up to $4^{\text {th }}$ order terms | 4 | 2 | 1 | 1,12 |
| 12. a) | Find the point on the sphere $x^{2}+y^{2}+z^{2}=1$ nearest to the point $(2,1,1)$. | 4 | 2 | 2 | 1,12 |
| b) | Expand $e^{x}$ Siny in powers of x and y as far as terms of third degree. | 4 | 2 | 2 | 1,12 |

13. a) Is the set $\{(1,-2,3),(2,3,1),(-1,3,2)\}$ a basis for $\mathcal{R}^{3}$ ?
b) If $\mathrm{S}=\left\{\alpha_{1}, \alpha_{2}, \alpha_{3}, \ldots . \alpha_{n}\right\}$ is a basis for a finite dimensional vector space V of dimension n , then show that every element of V can be uniquely expressed as a linear combination of the elements of $S$.
14. a) Let $T: R^{4} \rightarrow R^{3}$ defined by
$T(x, y, z, t)=(x-y+z+t, x+2 z-t, x+y+3 z-3 t) \quad$.Find
Range(T), Null space (T), rank T and Nullity T.
b) Let $T: R^{3} \rightarrow R^{2}$ be a linear transformation defined by
$T(x, y, z)=(3 x+2 y-4 z, x-5 y+3 z)$. Find the matrix of the linear Transformation $T$ relative to the bases $\{(1,1,1),(1,1,0),(1,0,0)\}$ and $\{(1,3),(2,5)$,$\} .$
15. a) From the basis $\{(3,4,0),(2,1,-1),(-2,1,3)\}$, using Gram-Schmidt orthogonalization process construct orthonormal basis in vector space $R^{3}(\mathrm{R})$.
b) Find the characteristic values and corresponding Characteristic vectors of the matrix $\left[\begin{array}{ccc}0 & -2 & -3 \\ -1 & 1 & -1 \\ 2 & 2 & 5\end{array}\right]$
16. a) Find the evolute of the curve $x^{2}=4 a y$
b) If $u=\operatorname{Sin}^{-1}(x-y) ; x=3 t ; y=4 t^{3}$ find total derivative $\frac{d u}{d t}$
17. Answer any two of the following:
a) Is the set of all 2 X 2 real matrices over the field of real numbers a Vector space, with respect to addition and multiplication of matrices and scalar multiplication of matrix .?
b) Let $T: U \rightarrow V$ be a linear transformation and $S=\left\{\alpha_{1}, \alpha_{2}, \alpha_{3}, \ldots, \alpha_{n}\right\}$ is a basis of U , then show that vectors $T\left(\alpha_{1}\right), T\left(\alpha_{2}\right), T\left(\alpha_{3}\right), \ldots . T\left(\alpha_{n}\right)$ generate Range of T .
c) Is the matrix $\left[\begin{array}{ccc}-6 & 7 & -2 \\ -6 & 7 & 0 \\ 3 & -3 & 5\end{array}\right]$ diagonalizable? Explain

M: Marks; L: Bloom's Taxonomy Level; CO; Course Outcome; PO: Programme Outcome

| i) | Blooms Taxonomy Level -1 | $11 \%$ |
| :---: | :--- | :--- |
| ii) | Blooms Taxonomy Level -2 | $52 \%$ |
| iii) | Blooms Taxonomy Level $-3 \& 4$ | $37 \%$ |



